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RESEARCH PROBLEMS

With Volume 36 of Discrete Mathematics, a Research Problem Section has been established. Problems in this section are intended to be research level problems rather than standard exercises. People wishing to submit such problems should send them (in duplicate) to:

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The following should be included: (1) The name of the person(s) who orginally posed the problem; (2) the name and address of a person willing to act as a correspondent; and (3) references and other pertinent information.

The Editorial Board of Discrete Mathematics invites readers to provide information about solutions, partial results and other pertinent items related to problems posed earlier, if possible indicating the source of the information, for example papers appearing in different journals, preprints, etc. This information will be passed along to readers from time to time in order to keep them appraised of the current status of various problems.

People wishing to provide information about problems that appeared earlier should write to Professor Alspach. People wishing to correspond on technical matters concerning a problem should write to the correspondent.

Problem 44. Posed by Aviezri S. Fraenkel and Yaacov Yesha.

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For a theory of last-player-winning *annihilation games* see [2–5]. In particular, Algorithm B of [5] computes the previous-player-winning, next-player-winning, and draw positions in $O(n^6)$ steps, where the given digraph has n vertices. Recently B.Z. Sonkin modified this algorithm, improving it to $O(n^5)$ steps. Can it be further improved? What is the lower bound for an arbitrary digraph?

By a *winning strategy* we mean the computation of an optimal next move from the present next-player-winning or draw position. For the concepts of winning strategy in the narrow sense (one which depends only on the current position) and in the wide sense (which depends also on some of the predecessors of the current position), see [6]. Since combinatorial games are finite, every combinatorial game has a strategy in the narrow sense. Normally there is therefore no need to bother about a strategy in the wide sense. However, for annihilation games, the only known *polynomial* strategy is a strategy in the wide sense. Is there a polynomial strategy in the narrow sense for annihilation games?

Recently first results on last-player-lose annihilation games played on acyclic graphs were given by Fergusion [1], but many problems remain. Nothing seems to be known on last-player-lose annihilation games played on cyclic graphs.

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Problem 45. Posed by Aviezri S. Fraenkel and Joseph Kahane.

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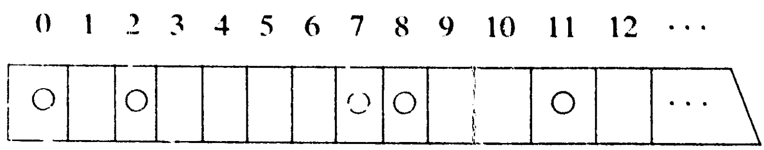
The game of *Welter* [5] is played by two players playing alternately on a semi-infinite strip whose squares are numbered 0, 1, 2, ... starting



from its end. A finite number m of (indistinguishable) coins is initially placed on distinct squares. A move consists of selecting a coin and moving it in the direction of the end to an *unoccupied* square. Play ends when all coins are jammed in positions $0, 1, \ldots, m-1$. The player first unable to move loses, his opponent wins. For a polynomial strategy see Berlekamp [1] and Conway [3, Ch. 13].

A theory is sought for Generalized Welter, that is, Welter played on an arbitrary digraph. However, any nontrivial generalization would be interesting at this stage. Some initial results were recently obtained on k-*Welter* which is played as Welter, except that each player can move a coin at most k squares in the direction of the end to an unoccupied square [4].

In a different direction, the game of antonim is played as Welter, except that square 0 has a hole, through which all incoming coins fall and disappear. This is nim in which the heaps are always distinct. Isabella Loimer (age 9) invented the game when she got frustrated with the 'traffic jams' of Welter. Her father Hermann Loimer subsequently found that the game is described in [2]. Also here only very simple facts seem to be known to date.

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Problem 46. Posed by Katherine Heinrich.

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An A-design of order n where n is odd is defined as follows:

(1) It is an $n \times n$ array containing the elements $1, 2, \ldots, n$.

(2) The main diagonal of the array consists of the elements $1, 2, \ldots, n$.

(3) If the element k occurs in the ith row (or column), $k \neq i$, then it occurs twice in that row (or column).

(4) The element i occurs exactly once in the ith row and ith column, that is, it occurs in the main diagonal position.

(5) The ith row and ith column between them contain each of the elements $1, 2, \ldots, n$ twice. We count i twice; once in the row and once in the column.

(6) When the array is superimposed on its transpose, every ordered pair (i, j), $i, j \in \{1, 2, \ldots, n\}$, occurs exactly once.

Various values of odd n for which A-designs exist are determined in [1] and using the results of [2] it is not difficult to show that for sufficiently large values of odd n A-designs always exist. Is it the case that A-designs exist for all odd orders greater than 3?

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